

Electronegativity Force Field for Prediction of Elastic Moduli

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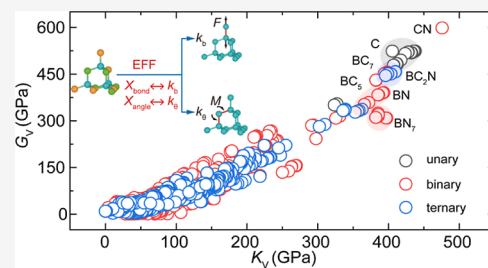
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ABSTRACT: Macroscopic elastic moduli (*i.e.*, bulk modulus and shear modulus) of covalent crystals are mainly determined by microscopic structures and stiffnesses. Herein, the microscopic bond and angle force constants of covalent crystals were parameterized from their atomic electronegativities, which is named the electronegativity force field (EFF). Based on this force field, the elastic moduli of covalent crystals can be directly obtained by molecular mechanics calculations. The calculated moduli for various covalent crystals are generally consistent with first-principles calculations, while the computational cost is reduced by several orders of magnitude, indicating the accuracy and efficiency of the EFF. Finally, we found 25 ultrahigh-modulus crystals with a bulk modulus greater than 350 GPa, which demonstrates that this force field can be used for screening of ultrahigh-modulus materials from numerous crystal candidates.



INTRODUCTION

Elastic moduli (*i.e.*, bulk modulus and shear modulus) not only measure the resistance to elastic deformation but are also closely related to other important properties of a material. For example, the bulk modulus has been used as an indication to screen superhard materials since a high bulk modulus is generally essential for superhard crystals.¹ High-modulus materials have been used in a wide range of scientific and industrial applications^{2–4} from high-pressure science to precision machining.

Driven by the increasing demand for high-modulus materials, several approaches have been developed to determine the elastic moduli of materials, including experimental measurements,⁵ first-principles calculations,^{6–8} classical molecular dynamics simulations,^{9–11} empirical/semi-empirical formulae,^{12–18} and machine learning prediction.¹⁹ Using these approaches, remarkable achievements have been made, and a few high-modulus materials with a bulk modulus rivaling or exceeding that of diamond, such as C₃N₄ (425–496 GPa),⁷ BN (400 GPa),^{20,21} ReC (422 GPa),²² and lonsdaleite (437 GPa),^{23,24} have been reported. However, as the number of crystal structures identified from experiments and computations in material databases [*e.g.*, the Crystallography Open Database (COD),²⁵ Inorganic Crystal Structure Database (ICSD),²⁶ Materials Project (MP),²⁷ and Open Quantum Materials Database (OQMD)²⁸] grows rapidly, it becomes challenging to determine elastic moduli using the above-mentioned approaches. This is because classical molecular dynamics and empirical/semi-empirical formulae are only applicable for specific elemental combinations with limited accuracy, while experimental measurements and first-principles calculations are too expensive and time-consuming. For example, after decades of calculations, only about 10⁴ crystals in the Materials Project²⁷ (on the order of 1% of all known

crystal structures, as shown in Table S1) have first-principles-calculated elastic constants. Hence, approaches for determining the elastic moduli of materials that can balance accuracy and efficiency are needed.

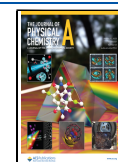
Macroscopic elastic moduli of covalent crystals are mainly determined by microscopic bond and angle stiffnesses^{29,30} that essentially originate from elemental combinations. Hence, it is possible to estimate the elastic moduli of a material solely from its elemental combinations. As pioneers, Rappe *et al.*³¹ developed a universal force field (UFF) in which the force field parameters for the full periodic table of elements are estimated using general rules based only on the element, its hybridization, and its connectivity. However, the calculated moduli using the UFF are far from first-principles calculations, as demonstrated in our following investigation. Li and Xue proposed electronegativity scales for ionic³² and covalent³³ crystals, which have been used in empirical formulae for predicting the bulk modulus¹⁵ and hardness^{14,33} for specific covalent crystals. The usefulness of these formulae indicates the feasibility of predicting macroscopic properties from elemental combinations.

In the present work, we developed an electronegativity force field (EFF) in which the bond and angle force constants of covalent crystals were determined from the atomic electronegativities. The calculated bulk and shear moduli of unary, binary, and ternary covalent crystals using this force field are well consistent with those of first-principles calculations, while

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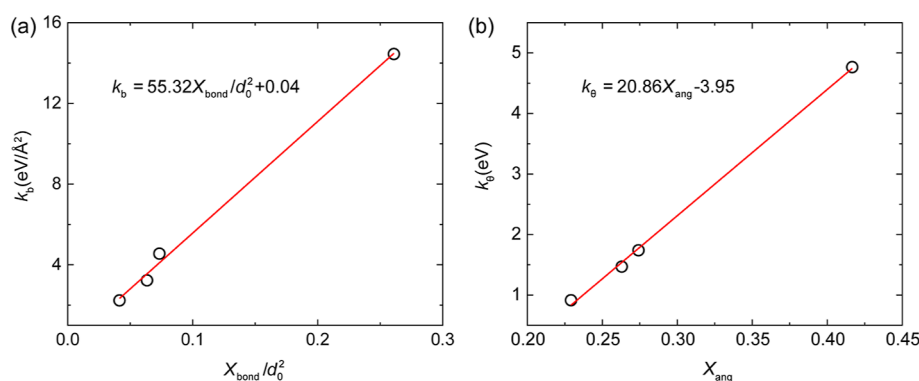


Figure 1. (a) Relationship between k_b and X_{bond}/d_0^2 and (b) relationship between k_θ and X_{ang} for nonpolar covalent crystals.

the computational cost is reduced by several orders of magnitude. These results demonstrate the accuracy and efficiency of the EFF. Finally, the EFF was used for screening ultrahigh-modulus materials from known crystal structures, which accelerated the discovery of 25 ultrahigh-modulus crystals with a bulk modulus greater than 350 GPa.

METHODS

The present goal is to predict the bulk and shear moduli of covalent crystals from their elemental combinations. Considering that the macroscopic bulk and shear moduli of covalent crystals are mainly from the contribution of bond stretching and angle bending, we first established formulae between bond and angle force constants and atomic electronegativities, named the EFF. By using the EFF, the elastic constants of covalent crystals can be calculated by using molecular mechanics methods. From the calculated elastic constants, the bulk and shear moduli can be extracted.

In addition to bond stretching and angle bending, we also explored the effects of dihedral and vdW interactions. Our calculations show that the differences between bulk and shear moduli of diamond with and without considering dihedral and vdW interactions are less than 10%. These results indicate that the macroscopic strain energy is mainly contributed by microscopic energies of bond stretching and angle bending, which can be described by $V_b = k_b(d - d_0)^2$ and $V_a = k_\theta(\theta - \theta_0)^2$, respectively, where the parameters of d_0 , θ_0 , k_b , and k_θ are the equilibrium bond length, equilibrium angle, bond force constant, and angle force constant, respectively. Since d_0 and θ_0 can be obtained from the known crystal structure, we only need to determine k_b and k_θ in the following investigation. Electronegativity is an important parameter that scales the holding energy between atoms and valence electrons. The bond electronegativity is a measure of the resistance to the bond stretching,¹⁴ which can be defined as

$$X_{\text{bond}} = \sqrt{\frac{X_a \times X_b}{N_a \times N_b}} \quad (1)$$

for a bond formed between atoms a and b, where X_a (N_a) and X_b (N_b) are the electronegativities³³ (connected bond numbers or coordination number) of atoms a and b, respectively. Similarly, the angle electronegativity is a measure of the resistance to the angle bending, which can be defined as

$$X_{\text{ang}} = \sqrt[3]{\frac{X_a \times X_b \times X_c}{N_{aa} \times N_{ab} \times N_{ac}}} \quad (2)$$

for an angle formed among atoms a, b, and c, where X_a (N_{aa}), X_b (N_{ab}), and X_c (N_{ac}) are the electronegativities (connected angle numbers) of atoms a, b, and c, respectively. For nonpolar covalent crystals, we found that k_b and k_θ are linear to X_{bond}/d_0^2 and X_{ang} , respectively (Figure 1). By fitting the known data for four typical nonpolar covalent crystals [C, Si, Ge, and Sn (α -Sn)], the formulae for the prediction of the bond and angle force constants from the electronegativities can be written as

$$k_b = \frac{55.32X_{\text{bond}}}{d_0^2} + 0.04 \quad (3)$$

$$k_\theta = 20.86X_{\text{ang}} - 3.95 \quad (4)$$

where the units of k_b , d_0 , and k_θ are $\text{eV}/\text{\AA}^2$, \AA , and eV , respectively. It should be noted that the existing accurate force fields are usually limited to a narrow range of elements, such as B, C, N, and O. Our work provides a fast and universal approach to determine the force constants from the atomic electronegativities. Meanwhile, taking diamond as an example, we compared our force field with other well-known force fields in the literature^{31,34–36} (Table S2). These results indicate that our force field is generally more accurate than previous force fields when comparing with density functional theory (DFT) calculations.

It should be noted that eqs 3 and 4 were developed for nonpolar covalent crystals (C, Si, Ge, and α -Sn). For polar covalent crystals, the difference of the electronegativities of atoms in chemical bonds and angles will destroy the uniform distribution of valence electrons, which will weaken the resistances to the bond stretching and the angle bending. As a result, the ionicity of polar covalent crystals would reduce the bond and angle force constants because of the asymmetry distribution of valence electrons in chemical bonds and angles. Hence, the predicted moduli for polar covalent crystals using the above formulae (eqs 3 and 4) are generally overestimated compared with those from first-principles calculations (Figure S1). To address this issue, we proposed the bond ionicity (f_b) and angle ionicity (f_a) to take the ionicity-induced reduction of force constants into account.

$$f_b = \frac{|X_a - X_b|}{X_a + X_b} \quad (5)$$

$$f_a = \frac{|X_a - X_b|}{X_a + X_b} + \frac{|X_a - X_c|}{X_a + X_c} \quad (6)$$

From the bond ionicity and angle ionicity, eqs 3 and 4 were modified by introducing the exponential loss functions of f_b^2

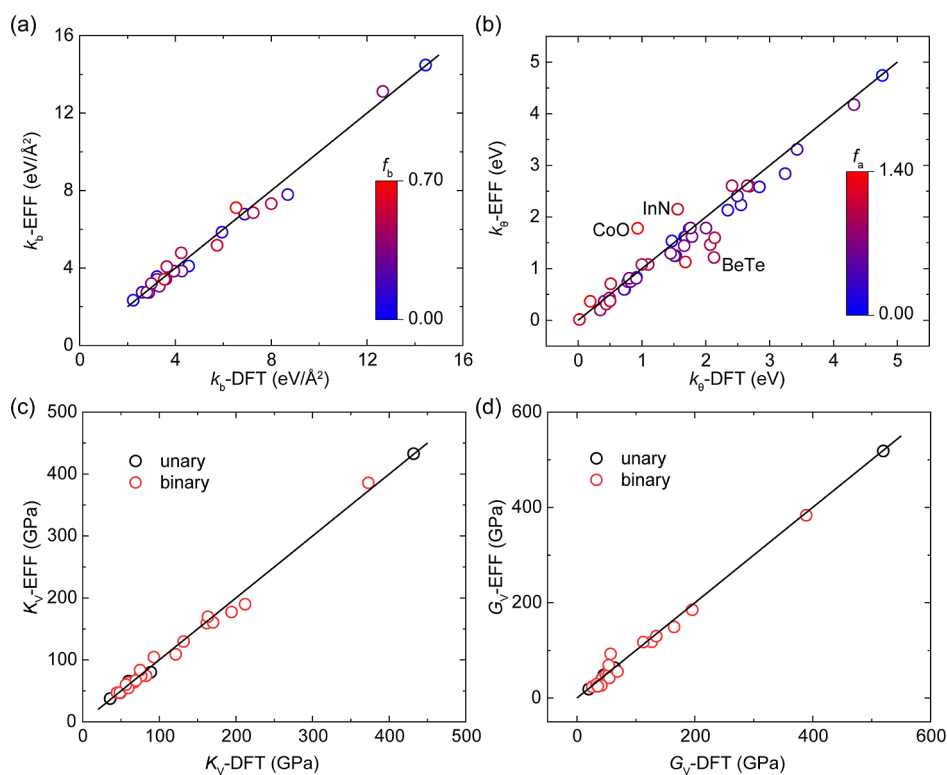


Figure 2. (a) Bond force constants and (b) angle force constants for covalent crystals from eqs 7 and 8 as compared with those from first-principles calculations. (c) Bulk moduli and (d) shear moduli for covalent crystals calculated based on the EFF as compared with first-principles calculations. The raw training data set is summarized in Table S3.

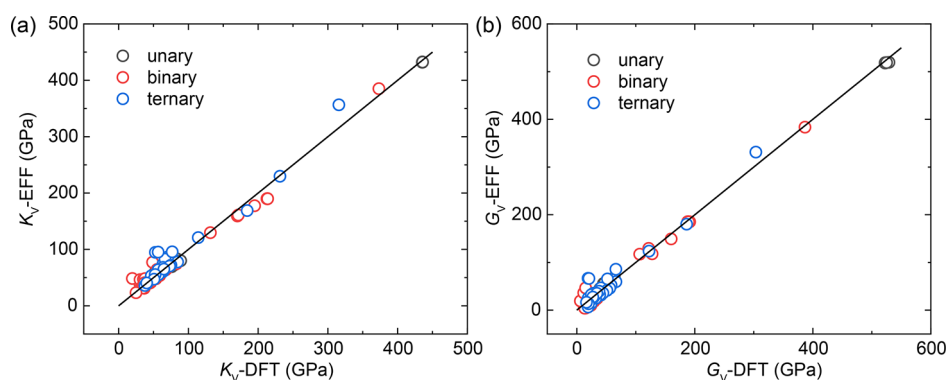


Figure 3. (a) Predicted bulk moduli and (b) shear moduli based on the EFF as compared with corresponding first-principles-calculated moduli. The test data set is not overlapped with the previous training data set in Figure 2.

and f_a^2 . Similar exponential loss functions have also been adopted to predict the mechanical properties of covalent crystals in previous works.^{14,15} By fitting the data for 25 unary and binary crystals from DFT calculations (see Supporting Information for this training data set), the fitting formulae for predicting the bond and angle force constants of polar covalent crystals were generalized as

$$k_b = \left(\frac{55.32X_{\text{bond}}}{d_0^2} + 0.04 \right) e^{-0.16f_b^2} \quad (7)$$

$$k_\theta = (20.86X_{\text{ang}} - 3.95) e^{-0.51f_a^2} \quad (8)$$

Here, d_0 is adopted as the equilibrium bond length in the crystal. For these unary and binary crystals, there is only one type of bond. As shown in Figure 2, the force constants and

elastic moduli for polar covalent crystals predicted by these generalized formulae are consistent with first-principles calculations. To further quantify the accuracy of the fitting results with respect to first-principles calculations, the mean error (ME), mean absolute error (MAE), mean relative error (MRE), and mean absolute relative error (MARE) were calculated. The calculated ME, MAE, MRE, and MARE for the bulk (shear) modulus based on the EFF are -1.81 (-2.92) GPa, 6.32 (7.57) GPa, -1.05 (-5.02)%, and 5.99 (13.06)%, respectively. These results indicate that the fitting results of the EFF are in good agreement with those of first-principles calculations.

RESULTS AND DISCUSSION

Based on the atomic electronegativities, eqs 7 and 8 can be used to predict the force constants of covalent crystals. Based

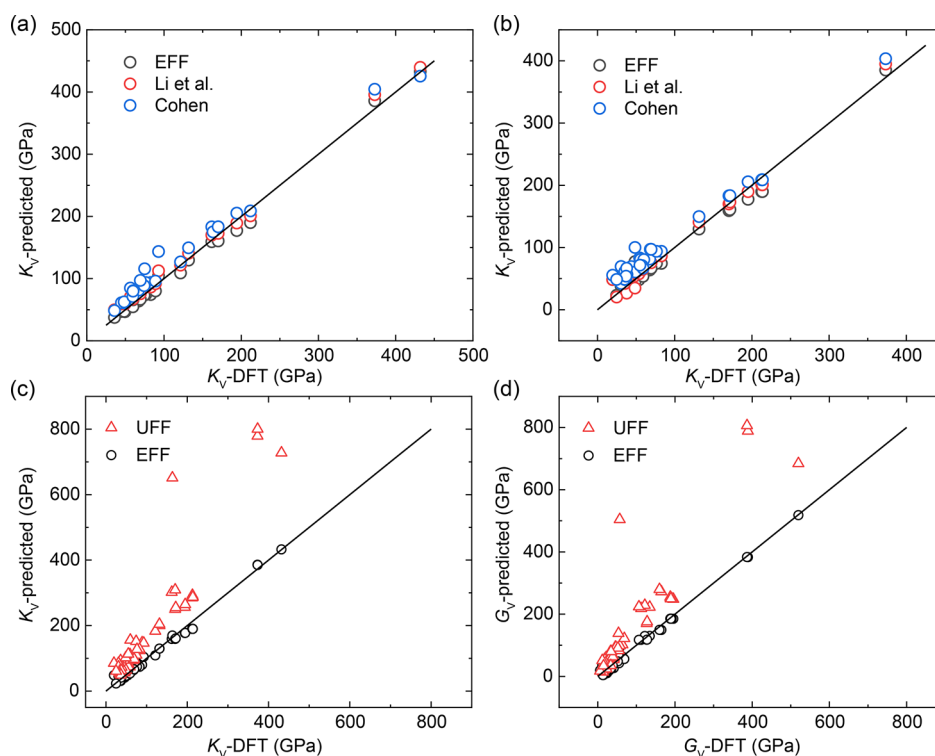


Figure 4. (a) Predicted bulk moduli of 25 crystals from the Materials Project based on the EFF, Li *et al.*,¹⁵ and Cohen¹³ as compared with those from first-principles calculations. (b) Predicted bulk moduli of 63 binary covalent crystals from the Materials Project based on the EFF, Li *et al.*,¹⁵ and Cohen¹³ as compared with those from first-principles calculations. (c) Bulk and (d) shear moduli of 88 covalent crystals predicted based on the EFF and the UFF as compared with first-principles calculations.

on the EFF, the elastic constants of known crystal structures can be determined by molecular mechanics calculations. These calculations were performed using the large-scale atomic/molecular massively parallel simulator (LAMMPS),³⁷ in which the elastic constants are calculated by strain–stress methods.³⁸

To validate the accuracy of the prediction based on the EFF, we did tests on 110 covalent crystals (test data set that is not overlapped with the previous training data set), including 13 unary crystals, 63 binary crystals, and 34 ternary crystals. From the EFF-predicted force constants and crystal structures, the elastic moduli were calculated by molecular mechanics calculations (Figure 3 and Tables S4–S6). In general, the predicted moduli based on the EFF are in good agreement with those based on first-principles calculations. More specifically, the EFF model has demonstrated accurate predictions for tetrahedral covalent crystals with a Pauling electronegativity difference between the two atoms of less than 1.7³⁹ (Figure S3). To further quantify the accuracy, we calculated the ME, MAE, MRE, and MARE for the predicted bulk (shear) moduli based on the EFF with respect to first-principles calculations as -1.79 (-1.16) GPa, 8.85 (7.58) GPa, 2.57 (0.54)%, and 11.27 (28.28)%, respectively (Table S7). Considering that the deviations of elastic moduli between first-principles calculations from different functionals are about 10–20%,⁴⁰ these results indicate that the accuracy of the prediction based on the EFF falls within the range of intrinsic errors of first-principles calculations. Hence, the accuracy of the prediction based on the EFF is comparable to that of first-principles calculations. We also compared the predicted moduli based on the EFF with those based on empirical/semi-empirical formulae for estimation of the bulk moduli proposed by Cohen¹³ and Li *et al.*¹⁵ (Figure 4a,b). The calculated ME,

MAE, MRE, and MARE with respect to first-principles calculations (Table 1) indicate that the accuracy of the

Table 1. ME, MAE, MRE, and MARE of Bulk Moduli Predicted by the EFF, Li *et al.*,¹⁵ and Cohen¹³ with Respect to First-Principles Calculations^a

| | EFF | Li <i>et al.</i> | Cohen |
|---------------------|-------|------------------|-------|
| 25 fitting crystals | | | |
| ME (GPa) | −1.81 | 9.27 | 16.74 |
| MAE (GPa) | 6.32 | 10.56 | 17.51 |
| MRE (%) | −1.05 | 13.20 | 21.96 |
| MARE (%) | 5.99 | 13.82 | 22.20 |
| 63 binary crystals | | | |
| ME (GPa) | −5.53 | 3.46 | 14.85 |
| MAE (GPa) | 10.03 | 10.52 | 16.96 |
| MRE (%) | 0.44 | 13.50 | 32.66 |
| MARE (%) | 12.36 | 18.80 | 33.65 |

^aNote: 25 crystals in the training data set were used for comparison in this table. Unless otherwise noted, the elastic moduli were obtained from the Materials Project.

prediction based on the EFF is improved compared with those of Li *et al.*¹⁵ and Cohen.¹³ In addition to comparisons with empirical/semi-empirical formulae, we also compared the predicted moduli based on the EFF with those based on the widely used UFF.³¹ As shown in Figure 4c,d, the elastic moduli are generally overestimated by the UFF. The calculated ME, MAE, MRE, and MARE of the UFF with respect to first-principles calculations are all much higher than those of the EFF (Table S8). These results indicate that the accuracy of the

prediction based on the EFF is significantly improved compared with previous works.

The efficiency of the EFF was also explored by comparing the computational time for 25 crystals in the training data set (Figure S2). The average duration for calculating the elastic tensor of a crystal is less than 1 s with a single central processing unit, and the ratio of computational duration between first-principles calculations and calculations based on the EFF is as high as 3×10^5 . These results indicate that the computational cost of the EFF is largely reduced by over 5 orders of magnitude compared with that of first-principles calculations. To summarize, the predicted elastic moduli from the EFF are comparable to those from first-principles calculations, while the computational cost is neglectable, indicating the accuracy and efficiency of the EFF. Such accuracy and efficiency make the EFF hold great promise for screening ultrahigh-modulus materials from numerous crystal candidates.

Finally, we employed the EFF to discover high-modulus materials by prescreening numerous crystal candidates in the Materials Project (MP)²⁷ and the Open Quantum Materials Database (OQMD).²⁸ First, a total of 3139 tetrahedral crystals in the MP (870 crystals) and OQMD (2269 crystals) were selected. Afterward, molecular mechanics calculations based on the EFF were employed to predict the bulk and shear moduli of these crystals (Figure 5). From these predictions, we found

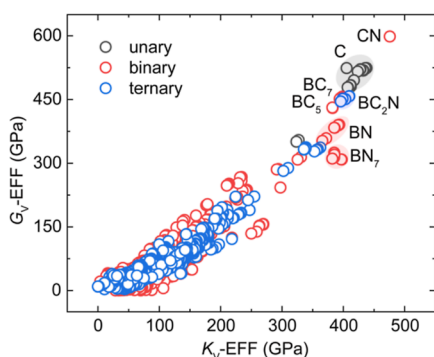


Figure 5. Predicted elastic moduli for 3139 crystal candidates in the Materials Project (MP)²⁷ and the Open Quantum Materials Database (OQMD).²⁸

39 unique crystals with a predicted bulk modulus over 300 GPa. The highest bulk modulus (475.7 GPa) and shear modulus (598.5 GPa) belonged to CN (OQMD-1218937). However, the first-principles calculations show that the elastic tensor of this crystal is not positive definite, indicating its mechanical instability. Finally, first-principles verification identified that 25 ultrahigh-modulus crystals meet the Born stability criteria⁴¹ and have a bulk modulus greater than 350 GPa (Table S9). The highest energy above hull for these crystals is 0.83 eV/atom, which is lower than the metastability threshold⁴² (Table S9). This implies that these 25 crystals are all promising for fabrication. We found that most high-modulus crystals are sp^3 carbon crystals and compounds of BN and BC. These results suggest that light elements B, C, and N with large values of electronegativity are good candidates to form high-modulus crystals.⁴³ Additionally, the above results indicate that the EFF performs well in predicting the bulk and shear moduli for covalent crystals. From the bulk and shear moduli, the hardness of a crystal can be estimated⁴⁴

$$H_V = \frac{0.55K_{\text{VRH}}G_{\text{VRH}}}{3K_{\text{VRH}} + G_{\text{VRH}}} \quad (9)$$

where the K_{VRH} and G_{VRH} are the Hill-averaged⁴⁵ bulk modulus and shear modulus, respectively. We calculated the hardness for 135 tetrahedral crystals and compared them with first-principles calculation results (Figure 6). It can be found

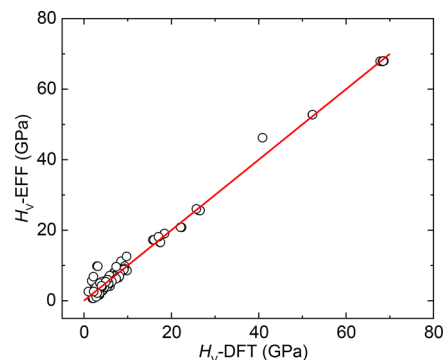


Figure 6. Predicted hardness for 135 crystals based on the EFF as compared with first-principles calculations.

that the hardness predicted by the EFF is also consistent with that predicted by first-principles calculations. This indicates that the EFF has a good performance in predicting the hardness of crystals.

CONCLUSIONS

In summary, we proposed an EFF to calculate the elastic moduli of covalent crystals in which the bond and angle force constants of covalent crystals were determined from the atomic electronegativities. Based on this force field, the elastic moduli of covalent crystals can be directly obtained by molecular mechanics calculations. The predicted bulk and shear moduli of unary, binary, and ternary covalent crystals using this force field from the EFF are comparable to those of first-principles calculations, while the computational cost is neglectable, indicating the accuracy and efficiency of the EFF. Finally, we used the EFF to screen ultrahigh-modulus materials from numerous crystal candidates and identified 25 ultrahigh-modulus crystals with a bulk modulus greater than 350 GPa. Additionally, the promising use of the EFF for predicting the hardness of crystals was discussed.

ASSOCIATED CONTENT

Supporting Information

The Supporting Information is available free of charge at <https://pubs.acs.org/doi/10.1021/acs.jpca.3c03173>.

Methods to parameterize the EFF and application of the EFF, and supplementary figures and tables (PDF)

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Notes

The authors declare no competing financial interest.

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Supporting Information for

Electronegativity Force Field for Prediction of Elastic Moduli

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This supporting information contains:

- (1) Methods to Parameterize the Electronegativity Force Field.**
- (2) Application of the Electronegativity Force Field.**
- (3) Figures S1-S3.**
- (4) Tables S1-S9.**

Methods to Parameterize the Electronegativity Force Field.

To show how to get the EFF model, we took the bond stiffness and angle stiffness of unary tetrahedral crystals of C, Si, Ge, and α -Sn as an example. First, we calculated elastic constants of C, Si, Ge, and α -Sn by first-principles calculations. The first-principles density functional theory (DFT) calculations were performed with the Vienna *Ab-Initio* Simulation Package (VASP).¹ The Perdew-Burke-Ernzerhof parameterization of the generalized gradient approximation was used for the exchange-correlation functional.² An energy cut-off of 520 eV was employed. A k -point mesh with a density about 40 Å (the product of each lattice constant and the corresponding number of k -points) was used in Brillouin zone sampling.³ The conjugate gradient algorithm was adopted for structural relaxation. The energy and the forces were converged to 1×10^{-8} eV/atom and 1×10^{-2} eV/Å, respectively. For these four unary covalent crystals, there is only one type of bond and angle. The bond and angle stiffnesses were obtained by fitting the first-principles calculated bulk and shear moduli⁴ converged to the absolute error of less than 1 GPa. Then, we took the bond stiffness and angle stiffnesses of crystal BN as an example. For the BN crystal, there are one type of bond and two types of angles. Therefore, we have one bond stiffness and two angle stiffnesses. The assumption was that the ratio of two angle stiffness equals the angle stiffness obtained from **eq 4**. By substituting the electronegativity⁵ and angle electronegativity into **eqs 2** and **4**, we obtained the ratio of angle stiffness of angle \angle BNB (3.350 eV) and \angle NBN (5.389 eV) as 1.609. Then, we changed the angle stiffness of \angle BNB, while the angle stiffness of \angle NBN was calculated by the product of the angle stiffness \angle BNB and angle stiffness ratio 1.609. Finally, we obtained the bond and angle stiffness as 12.659 eV/Å², 2.685 eV(\angle BNB), and 4.320 eV(\angle NBN), respectively.

Application of the Electronegativity Force Field.

To show how to use EFF model for obtaining the parameters of bond and angle stiffness, we took binary tetrahedral crystal BN as an example. In the BN crystal, there are 8 atoms in the conventional cell, 4 B atoms tetrahedrally bonded with N atoms. The bond electronegativity (X_{bond}) and bond ionicity (f_b) can be obtained by substituting the atom electronegativity⁵ into **eqs 1 and 5** as 0.594 and 0.354 respectively. From BN crystal structure, we can obtain the bond length (d_0) of BN, which is 1.570 Å. Then, the bond stiffness k_b for harmonic bond of B-N was obtained as 13.098 eV/Å² from **eq 7**.

There are two types of angles and 48 angles in the conventional cell, the ratio of angle numbers for $\angle\text{BNB}$ and $\angle\text{NBN}$ are 1/2 and 1/2, respectively. The angle electronegativity and angle ionicity of $\angle\text{BNB}$ ($\angle\text{NBN}$) can be obtained by substituting the atom electronegativity⁵ into **eqs 2 and 6** as 0.350 (0.448) and 0.707 (0.707), respectively. By substituting the angle electronegativity and angle ionicity (f_a) into **eq 8**, we obtained the harmonic angle stiffness k_θ of $\angle\text{BNB}$ and $\angle\text{NBN}$ as 2.595 eV and 4.176 eV, respectively. It should be noted that crystal structures were obtained from the database, and the elastic constants were obtained from molecular mechanics with LAMMPS.⁶ Then, the EFF predicted bulk modulus ($K_v\text{-EFF}$) and shear modulus ($G_v\text{-EFF}$) were obtained from elastic constants as 385.8 GPa and 383.2 GPa, respectively, which are close to the DFT results (372.9 GPa and 389.0 GPa).

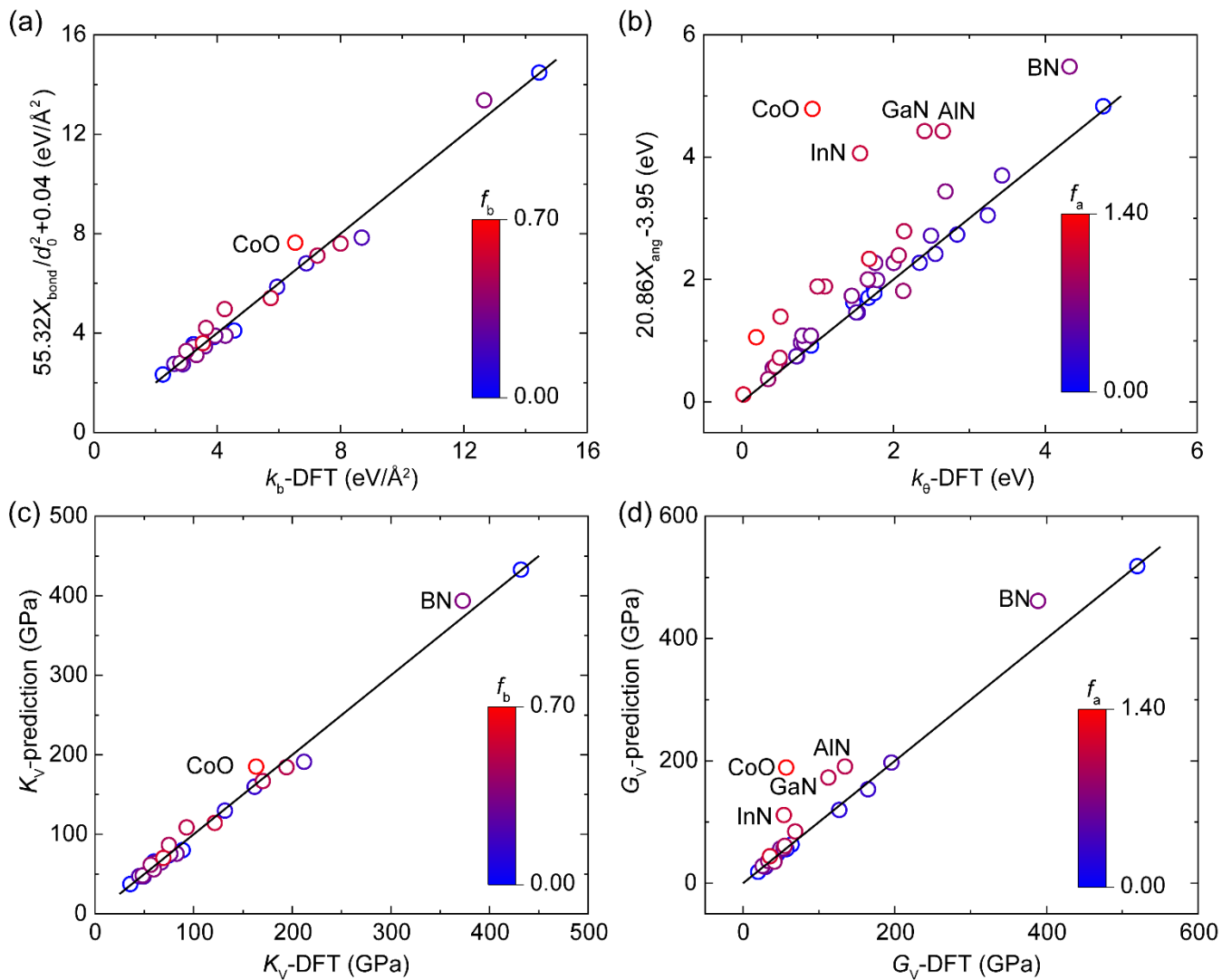


Figure S1. Predicted (a) bond force constants and (b) angle force constants for covalent crystals from eqs 3 and 4 as compared with first-principles calculations. (c) Bulk moduli and (d) shear moduli for covalent crystals calculated based on the eqs 3 and 4 as compared with first-principles calculations.

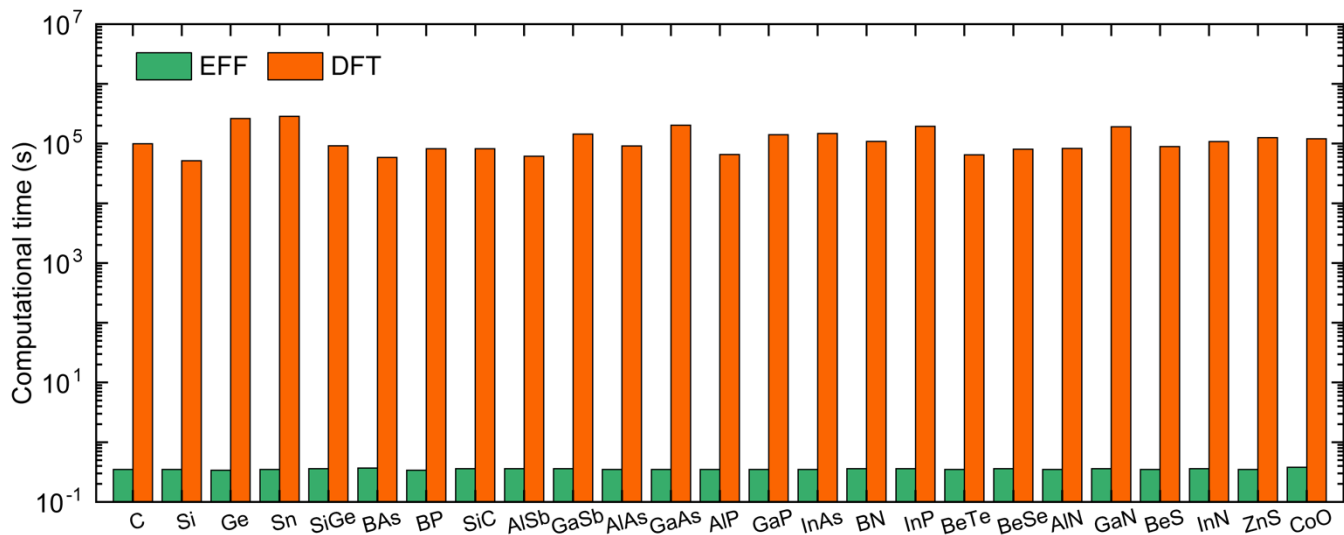


Figure S2. Computational time for elastic constants of 25 crystals in the training data set based on the EFF and DFT calculations.

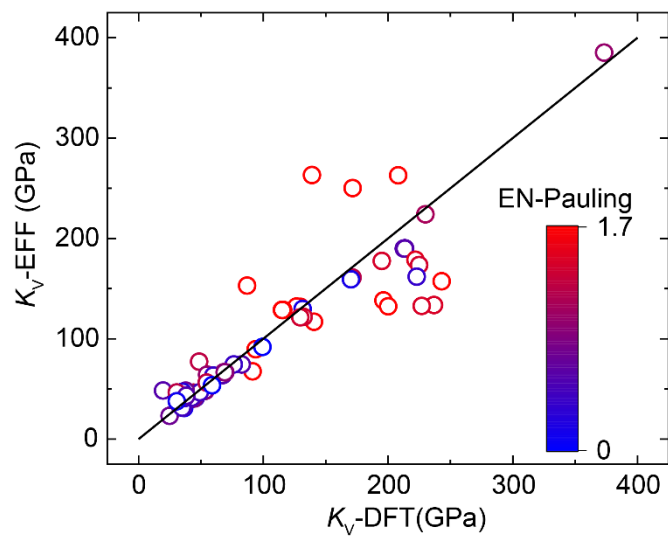


Figure S3. Predicted bulk moduli of 87 binary crystals from Material Project based on the EFF as compared with those from first-principles calculations. The crystals with Pauling electronegativity difference (EN-Pauling) between the two atoms larger than 1.7 are labeled with red color.

Table S1. Database statistics.

| Database | Crystals | Elastic constants | Ref |
|--|-----------|-------------------|-----|
| Cambridge Structural Database, CSD | 1,101,469 | 0 | 7 |
| Open Quantum Materials Database, OQMD | 815,654 | 0 | 8 |
| Crystallography Open Database, COD | 468,173 | 0 | 9 |
| Inorganic Crystal Structure Database, ICSD | 232,012 | 0 | 10 |
| Materials Project, MP | 131,613 | 13,172 | 11 |

Table S2. Diamond force field parameters and calculated bulk and shear moduli compared with well-known force field parameters in literature.¹²⁻¹⁵

| Force field | k_b (eV/Å ²) | k_θ (eV) | K_V (GPa) | G_V (GPa) |
|-----------------------|----------------------------|-----------------|-------------|-------------|
| DFT | | | 431.8 | 519.8 |
| EFF | 14.448 | 4.765 | 432.7 | 518.3 |
| UFF ^{12, 13} | 15.168 | 9.289 | 463.4 | 546.0 |
| CVFF ¹⁴ | 13.994 | 2.021 | 424.2 | 286.4 |
| PCFF ¹⁵ | 12.995 | 1.714 | 382.9 | 262.8 |

Table S3. Parameters related to calculations of bulk and shear moduli for 25 typical unary and binary tetrahedral crystals.

| Crystals | MP-ID | d (Å) | K_V (GPa) | G_V (GPa) | k_b -DFT (eV/Å ²) | k_0 -DFT (eV) | f_b | f_a | K_V -EFF (GPa) | G_V -EFF (GPa) |
|-----------------|-----------|---------|-------------|-------------|---------------------------------|-----------------|-------|-------|------------------|------------------|
| C | mp-66 | 1.547 | 431.8 | 519.8 | 14.448 | 4.765 | 0.000 | 0.000 | 432.7 | 518.3 |
| Si | mp-149 | 2.368 | 89.0 | 64.1 | 4.555 | 1.740 | 0.000 | 0.000 | 80.0 | 63.4 |
| Ge | mp-32 | 2.494 | 59.9 | 45.6 | 3.230 | 1.470 | 0.000 | 0.000 | 65.8 | 48.3 |
| Sn | mp-117 | 2.880 | 35.9 | 20.1 | 0.000 | 0.915 | 0.000 | 0.000 | 37.5 | 18.7 |
| SiGe | mp-978534 | 2.423 | 74.3 | 57.4 | 3.891 | 1.750 | 0.021 | 0.042 | 73.2 | 55.7 |
| BA _s | mp-10044 | 2.086 | 131.8 | 127.0 | 5.945 | 2.840 | 0.108 | 0.216 | 129.6 | 117.7 |
| BP | mp-1479 | 1.969 | 161.9 | 164.9 | 6.891 | 3.243 | 0.143 | 0.285 | 159.3 | 149.1 |
| SiC | mp-8062 | 1.896 | 212.0 | 195.7 | 8.693 | 3.430 | 0.206 | 0.413 | 189.9 | 185.0 |
| AlSb | mp-2624 | 2.699 | 49.5 | 30.0 | 2.887 | 1.530 | 0.214 | 0.427 | 46.7 | 24.9 |
| GaSb | mp-1156 | 2.692 | 44.9 | 29.2 | 2.611 | 1.510 | 0.214 | 0.427 | 47.0 | 25.1 |
| AlAs | mp-2172 | 2.483 | 67.2 | 41.6 | 3.607 | 1.690 | 0.280 | 0.561 | 63.8 | 39.5 |
| GaAs | mp-2534 | 2.490 | 60.1 | 42.1 | 3.235 | 1.784 | 0.280 | 0.561 | 63.3 | 39.2 |
| AlP | mp-1550 | 2.384 | 82.8 | 49.4 | 4.268 | 1.760 | 0.312 | 0.625 | 74.3 | 48.4 |
| GaP | mp-2490 | 2.384 | 76.4 | 53.5 | 3.939 | 2.003 | 0.312 | 0.625 | 74.3 | 48.4 |
| InAs | mp-20305 | 2.680 | 48.5 | 26.0 | 2.808 | 1.450 | 0.341 | 0.682 | 47.3 | 23.3 |
| BN | mp-1639 | 1.570 | 372.9 | 389.0 | 12.659 | 4.320 | 0.354 | 0.707 | 385.8 | 383.2 |
| InP | mp-20351 | 2.579 | 59.7 | 33.3 | 3.327 | 1.662 | 0.372 | 0.743 | 54.7 | 29.0 |
| BeTe | mp-252 | 2.455 | 56.5 | 41.5 | 2.998 | 2.129 | 0.413 | 0.826 | 60.0 | 26.4 |
| BeSe | mp-1541 | 2.245 | 75.0 | 55.1 | 3.640 | 2.070 | 0.472 | 0.944 | 83.6 | 42.2 |
| AlN | mp-1700 | 1.906 | 194.2 | 134.6 | 8.002 | 2.650 | 0.500 | 1.000 | 177.2 | 129.7 |
| GaN | mp-830 | 1.970 | 170.2 | 112.6 | 7.250 | 2.410 | 0.500 | 1.000 | 160.5 | 117.5 |
| BeS | mp-422 | 2.111 | 93.0 | 68.9 | 4.243 | 2.140 | 0.507 | 1.014 | 104.4 | 55.6 |
| InN | mp-22205 | 2.186 | 121.4 | 53.9 | 5.740 | 1.560 | 0.549 | 1.097 | 109.0 | 68.8 |
| ZnS | mp-10695 | 2.359 | 69.4 | 35.7 | 3.537 | 1.680 | 0.581 | 1.163 | 66.7 | 24.8 |
| CoO | mp-19128 | 1.848 | 163.5 | 57.1 | 6.429 | 0.932 | 0.690 | 1.379 | 169.6 | 92.7 |

Table S4. Parameters related to calculations of bulk and shear moduli for 13 unary crystals.

| Crystals | MP-ID | d (Å) | K_V -DFT (GPa) | G_V -DFT (GPa) | f_b | f_a | K_V -EFF (GPa) | G_V -EFF (GPa) |
|----------|------------|---------|------------------|------------------|-------|-------|------------------|------------------|
| Sn | mp-949028 | 2.872 | 31.7 | 17.8 | 0 | 0 | 33.0 | 16.3 |
| Ge | mp-128 | 2.528 | 52.3 | 39.4 | 0 | 0 | 59.4 | 45.3 |
| Ge | mp-1007760 | 2.489 | 59.2 | 46.7 | 0 | 0 | 66.2 | 48.4 |
| Si | mp-971662 | 2.375 | 76.3 | 45.2 | 0 | 0 | 69.4 | 55.7 |
| Si | mp-971661 | 2.371 | 79.3 | 47.0 | 0 | 0 | 72.2 | 55.7 |
| Si | mp-571520 | 2.383 | 80.6 | 62.5 | 0 | 0 | 75.0 | 60.7 |
| Si | mp-999200 | 2.373 | 82.1 | 47.7 | 0 | 0 | 75.2 | 57.6 |
| Si | mp-168 | 2.381 | 83.3 | 63.8 | 0 | 0 | 83.5 | 68.2 |
| Si | mp-165 | 2.368 | 89.0 | 63.0 | 0 | 0 | 80.1 | 63.3 |
| C | mp-47 | 1.549 | 435.4 | 529.1 | 0 | 0 | 431.7 | 519.5 |
| C | mp-616440 | 1.548 | 435.6 | 522.7 | 0 | 0 | 432.6 | 518.6 |
| C | mp-569567 | 1.548 | 435.6 | 522.5 | 0 | 0 | 432.0 | 518.0 |
| C | mp-611426 | 1.548 | 435.8 | 525.1 | 0 | 0 | 432.3 | 518.9 |

Table S5. Parameters related to calculations of bulk and shear moduli for 63 binary tetrahedral crystals.

| Crystals | MP-ID | d (Å) | K_v -DFT (GPa) | G_v -DFT (GPa) | f_b | f_a | K_v -EFF (GPa) | G_v -EFF (GPa) |
|----------|------------|---------|------------------|------------------|-------|-------|------------------|------------------|
| CdSe | mp-1070 | 2.690 | 44.4 | 15.3 | 0.509 | 1.019 | 40.5 | 9.2 |
| CdSe | mp-2691 | 2.690 | 45.0 | 16.8 | 0.509 | 1.019 | 40.5 | 9.5 |
| HgSe | mp-820 | 2.717 | 41.3 | 15.2 | 0.509 | 1.019 | 39.3 | 9.2 |
| HgS | mp-1123 | 2.601 | 49.3 | 17.6 | 0.509 | 1.019 | 46.6 | 13.1 |
| CdS | mp-2469 | 2.572 | 53.3 | 19.4 | 0.509 | 1.019 | 48.1 | 13.6 |
| CdS | mp-672 | 2.573 | 53.7 | 17.8 | 0.509 | 1.019 | 48.1 | 13.2 |
| ZnTe | mp-2176 | 2.678 | 46.0 | 24.0 | 0.463 | 0.925 | 41.2 | 10.0 |
| ZnTe | mp-8884 | 2.680 | 42.8 | 22.1 | 0.463 | 0.925 | 41.2 | 9.8 |
| MgSe | mp-13031 | 2.598 | 44.0 | 17.4 | 0.488 | 0.976 | 46.4 | 12.9 |
| MgS | mp-13032 | 2.467 | 54.7 | 22.0 | 0.488 | 0.976 | 56.2 | 18.2 |
| ZnSe | mp-1190 | 2.486 | 58.7 | 29.2 | 0.463 | 0.925 | 54.8 | 18.2 |
| ZnSe | mp-380 | 2.488 | 55.7 | 26.8 | 0.463 | 0.925 | 54.7 | 17.7 |
| AlAs | mp-8881 | 2.483 | 67.7 | 40.0 | 0.271 | 0.543 | 63.8 | 39.2 |
| AlN | mp-661 | 1.905 | 195.0 | 122.0 | 0.271 | 0.543 | 177.5 | 129.5 |
| AlP | mp-8880 | 2.384 | 82.8 | 47.1 | 0.271 | 0.543 | 74.3 | 48.1 |
| AlSb | mp-1018100 | 2.699 | 49.1 | 28.9 | 0.271 | 0.543 | 46.7 | 24.6 |
| BAs | mp-984718 | 2.086 | 131.6 | 127.7 | 0.099 | 0.197 | 129.6 | 117.9 |
| BN | mp-2653 | 1.571 | 373.2 | 386.7 | 0.099 | 0.197 | 385.2 | 383.5 |
| BP | mp-1008559 | 1.969 | 170.3 | 160.0 | 0.099 | 0.197 | 159.2 | 149.2 |
| GaAs | mp-8883 | 2.490 | 60.0 | 40.8 | 0.271 | 0.543 | 63.3 | 38.9 |
| GaN | mp-804 | 1.968 | 171.6 | 106.7 | 0.271 | 0.543 | 160.9 | 117.3 |
| GaP | mp-8882 | 2.384 | 76.4 | 52.3 | 0.271 | 0.543 | 74.4 | 48.1 |
| InP | mp-966800 | 2.582 | 57.8 | 31.7 | 0.332 | 0.665 | 54.5 | 28.5 |
| SiC | mp-7140 | 1.896 | 212.7 | 188.0 | 0.097 | 0.195 | 189.9 | 185.4 |
| SiC | mp-568619 | 1.896 | 213.1 | 191.5 | 0.097 | 0.195 | 189.9 | 185.1 |
| SiC | mp-9947 | 1.896 | 213.1 | 189.3 | 0.097 | 0.195 | 189.9 | 185.1 |
| SiC | mp-568656 | 1.896 | 213.1 | 189.1 | 0.097 | 0.195 | 190.0 | 185.2 |
| SiC | mp-568735 | 1.896 | 213.1 | 189.1 | 0.097 | 0.195 | 190.0 | 185.3 |
| SiC | mp-7631 | 1.896 | 213.2 | 188.9 | 0.097 | 0.195 | 189.9 | 185.1 |
| SiC | mp-568696 | 1.896 | 213.2 | 189.3 | 0.097 | 0.195 | 190.0 | 185.1 |
| SiC | mp-570690 | 1.897 | 213.3 | 189.2 | 0.097 | 0.195 | 189.8 | 185.0 |
| SiC | mp-582034 | 1.896 | 213.3 | 189.4 | 0.097 | 0.195 | 190.0 | 185.1 |
| SiC | mp-570641 | 1.897 | 213.4 | 189.7 | 0.097 | 0.195 | 189.8 | 184.9 |
| SiC | mp-11714 | 1.896 | 213.4 | 189.1 | 0.097 | 0.195 | 190.0 | 185.2 |

| | | | | | | | | |
|------|------------|-------|-------|-------|-------|-------|-------|-------|
| SiC | mp-570791 | 1.896 | 213.6 | 188.9 | 0.097 | 0.195 | 189.9 | 185.2 |
| SiC | mp-570985 | 1.896 | 213.6 | 189.3 | 0.097 | 0.195 | 190.0 | 185.2 |
| SiC | mp-567551 | 1.896 | 213.6 | 189.3 | 0.097 | 0.195 | 190.1 | 185.3 |
| SiC | mp-567505 | 1.897 | 213.7 | 189.4 | 0.097 | 0.195 | 189.8 | 185.0 |
| SiSn | mp-1009813 | 2.628 | 59.0 | 39.0 | 0.097 | 0.195 | 53.6 | 35.0 |
| ZnS | mp-560588 | 2.361 | 68.4 | 33.1 | 0.463 | 0.925 | 66.5 | 24.2 |
| ZnS | mp-554405 | 2.361 | 69.1 | 33.5 | 0.463 | 0.925 | 66.6 | 24.2 |
| ZnS | mp-555763 | 2.360 | 69.2 | 33.3 | 0.463 | 0.925 | 66.6 | 24.3 |
| ZnS | mp-555280 | 2.361 | 69.4 | 33.7 | 0.463 | 0.925 | 66.5 | 24.2 |
| ZnS | mp-9946 | 2.361 | 69.4 | 33.9 | 0.463 | 0.925 | 66.5 | 24.2 |
| ZnS | mp-10281 | 2.361 | 69.6 | 33.5 | 0.463 | 0.925 | 66.5 | 24.2 |
| ZnS | mp-13456 | 2.361 | 69.6 | 33.7 | 0.463 | 0.925 | 66.5 | 24.2 |
| ZnS | mp-561258 | 2.361 | 69.6 | 34.3 | 0.463 | 0.925 | 66.5 | 24.2 |
| TlAs | mp-1007770 | 2.754 | 38.3 | 20.0 | 0.341 | 0.683 | 43.2 | 20.4 |
| SnSb | mp-16365 | 2.996 | 30.7 | 20.2 | 0.185 | 0.370 | 37.6 | 26.3 |
| MnTe | mp-1009222 | 2.769 | 19.7 | 16.0 | 0.257 | 0.513 | 48.4 | 32.8 |
| MgTe | mp-1039 | 2.821 | 34.4 | 12.5 | 0.488 | 0.976 | 34.1 | 6.0 |
| MgTe | mp-13033 | 2.820 | 32.7 | 13.6 | 0.488 | 0.976 | 34.1 | 6.2 |
| MgSe | mp-1018040 | 2.597 | 30.7 | 20.6 | 0.488 | 0.976 | 46.4 | 12.6 |
| InSb | mp-20012 | 2.872 | 35.0 | 20.4 | 0.332 | 0.665 | 36.1 | 14.2 |
| HgTe | mp-2730 | 2.884 | 36.7 | 13.6 | 0.509 | 1.019 | 30.9 | 4.0 |
| GaSb | mp-1018059 | 2.691 | 35.4 | 33.7 | 0.271 | 0.543 | 47.1 | 24.8 |
| CuI | mp-673245 | 2.628 | 37.9 | 16.1 | 0.438 | 0.876 | 48.2 | 19.4 |
| CdTe | mp-406 | 2.870 | 35.7 | 14.8 | 0.509 | 1.019 | 31.4 | 4.1 |
| CdTe | mp-12779 | 2.872 | 35.1 | 13.5 | 0.509 | 1.019 | 31.3 | 4.0 |
| AlBi | mp-1018132 | 2.792 | 37.0 | 22.8 | 0.271 | 0.543 | 40.8 | 19.5 |
| AgI | mp-22894 | 2.875 | 24.9 | 6.1 | 0.726 | 1.453 | 23.1 | 19.2 |
| CuCl | mp-22914 | 2.340 | 48.7 | 11.8 | 0.438 | 0.876 | 77.2 | 36.2 |
| MnSe | mp-2293 | 2.575 | 55.0 | 15.0 | 0.257 | 0.513 | 64.2 | 46.4 |

Table S6. Bulk and shear moduli of 34 ternary tetrahedral crystals calculated by DFT and EFF.

| Crystals | MP-ID | K_v -DFT (GPa) | K_v -EFF (GPa) | G_v -DFT (GPa) | G_v -EFF (GPa) |
|---------------------|------------|---------------------|---------------------|---------------------|---------------------|
| BeGeP ₂ | mp-1013527 | 71.8 | 87.6 | 66.5 | 59.5 |
| BeGeAs ₂ | mp-1009088 | 63.7 | 74.2 | 53.4 | 47.9 |
| BeSiAs ₂ | mp-1009087 | 73.6 | 79.0 | 58.3 | 52.2 |
| CuBS ₂ | mp-12954 | 114.1 | 121.0 | 66.3 | 85.6 |
| AlCuS ₂ | mp-4979 | 66.6 | 82.8 | 40.9 | 46.6 |
| ZnSiP ₂ | mp-4763 | 84.6 | 77.7 | 55.9 | 45.9 |
| ZnGeP ₂ | mp-4524 | 75.0 | 72.1 | 51.2 | 41.3 |
| ZnSnP ₂ | mp-4175 | 63.8 | 60.9 | 40.9 | 31.4 |
| AlCuSe ₂ | mp-8016 | 62.9 | 67.0 | 30.3 | 35.9 |
| GaCuSe ₂ | mp-4840 | 59.7 | 65.7 | 30.4 | 35.2 |
| ZnSiAs ₂ | mp-3595 | 66.1 | 65.2 | 44.3 | 35.8 |
| ZnGeAs ₂ | mp-4008 | 60.6 | 60.8 | 38.9 | 32.2 |
| ZnSnAs ₂ | mp-5190 | 52.4 | 52.1 | 32.1 | 24.5 |
| InCuS ₂ | mp-22736 | 63.7 | 68.2 | 27.8 | 35.0 |
| MgSiP ₂ | mp-2961 | 73.0 | 70.3 | 37.8 | 39.5 |
| MgSiAs ₂ | mp-1016197 | 61.6 | 59.7 | 30.9 | 30.8 |
| InCuSe ₂ | mp-22811 | 53.0 | 56.0 | 23.2 | 27.0 |
| AlCuTe ₂ | mp-8017 | 51.6 | 49.7 | 25.9 | 23.4 |
| ZnSnSb ₂ | mp-4756 | 40.4 | 39.3 | 24.3 | 13.5 |
| GaCuTe ₂ | mp-3839 | 47.7 | 49.4 | 26.0 | 23.2 |
| TlCuSe ₂ | mp-14090 | 46.8 | 53.1 | 19.5 | 25.2 |
| BeSiN ₂ | mp-15704 | 231.3 | 229.7 | 186.2 | 180.1 |
| TlCuS ₂ | mp-14089 | 56.7 | 63.8 | 24.6 | 32.3 |
| ZnCdTe ₂ | mp-971837 | 37.8 | 35.9 | 19.3 | 6.5 |
| BeCN ₂ | mp-15703 | 315.8 | 356.4 | 303.5 | 331.3 |
| MgSnP ₂ | mp-1009083 | 57.0 | 55.0 | 28.4 | 26.3 |
| MgGeP ₂ | mp-34903 | 64.9 | 65.0 | 34.1 | 35.2 |
| CuBSe ₂ | mp-983565 | 76.8 | 95.7 | 52.1 | 65.3 |
| MgGeAs ₂ | mp-1016200 | 52.3 | 55.3 | 27.2 | 27.4 |
| ZnCdSe ₂ | mp-1017534 | 50.8 | 47.1 | 19.9 | 13.3 |

| | | | | | |
|----------------------------------|------------|-------|-------|-------|-------|
| AlGa ₃ N ₂ | mp-1008556 | 184.3 | 168.8 | 122.6 | 123.7 |
| FeCuS ₂ | mp-640073 | 52.7 | 94.8 | 19.2 | 66.6 |
| FeCuS ₂ | mp-3497 | 56.7 | 95.3 | 20.6 | 66.2 |
| CrCdTe ₂ | mp-1018083 | 40.2 | 40.4 | 16.5 | 16.4 |

Table S7. Mean error (ME), mean absolute error (MAE), mean relative error (MRE), and mean absolute relative error (MARE) of bulk and shear moduli for 110 covalent crystals (including 13 unary crystals, 63 binary crystals, and 34 ternary crystals) predicted by the EFF with respect to first-principles calculations.

| | K_V -EFF | G_V -EFF |
|-----------|------------|------------|
| ME (GPa) | -1.79 | -1.16 |
| MAE (GPa) | 8.85 | 7.58 |
| MRE (%) | 2.57 | 0.54 |
| MARE (%) | 11.27 | 28.28 |

Table S8. Mean error (ME), mean absolute error (MAE), mean relative error (MRE), and mean absolute relative error (MARE) of bulk moduli and shear moduli for 88 covalent crystals (including 25 fitting covalent crystals and 63 binary crystals) predicted by EFF and UFF^{12, 13} with respect to first-principles calculations.

| | K_V -EFF | K_V -UFF | G_V -EFF | G_V -UFF |
|-----------|------------|------------|------------|------------|
| ME (GPa) | -4.48 | 63.98 | -3.37 | 53.78 |
| MAE (GPa) | 8.97 | 63.98 | 7.30 | 53.78 |
| MRE (%) | 0.01 | 65.62 | -5.71 | 88.93 |
| MARE (%) | 10.55 | 65.62 | 26.29 | 88.93 |

Table S9. 25 covalent crystals with a bulk modulus over 350 GPa predicted by EFF and verified by first-principles calculations.

| MP/OQMD ID | Crystals | K_V -EFF (GPa) | K_V -DFT (GPa) | Energy above hull (eV/atom) |
|-------------|-------------------|------------------|------------------|-----------------------------|
| mp-1078845 | C | 417.3 | 413.9 | 0.27 |
| mp-1080826 | C | 407.5 | 402.0 | 0.30 |
| mp-1190171 | C | 407.0 | 401.1 | 0.29 |
| oqmd-637353 | C | 412.2 | 405.3 | 0.32 |
| mp-47 | C | 431.7 | 432.7 | 0.16 |
| mp-569517 | C | 432.6 | 432.6 | 0.14 |
| mp-569567 | C | 432.0 | 432.6 | 0.14 |
| mp-611426 | C | 432.3 | 432.7 | 0.15 |
| mp-611448 | C | 432.5 | 432.6 | 0.14 |
| mp-616440 | C | 432.6 | 432.5 | 0.14 |
| mp-66 | C | 432.7 | 431.8 | 0.14 |
| oqmd-603140 | C | 427.9 | 385.3 | 0.83 |
| mp-13151 | BN | 365.9 | 351.1 | 0.18 |
| mp-1639 | BN | 385.8 | 372.9 | 0.08 |
| mp-2653 | BN | 385.2 | 373.2 | 0.09 |
| mp-1095030 | BC ₇ | 395.1 | 389.8 | 0.28 |
| mp-1079046 | BC ₇ | 394.7 | 390.7 | 0.24 |
| mp-1078935 | BC ₇ | 394.3 | 392.9 | 0.21 |
| mp-1079661 | BC ₇ | 394.0 | 393.5 | 0.24 |
| mp-1080030 | BC ₇ | 394.3 | 393.3 | 0.24 |
| mp-1077125 | BC ₅ | 382.4 | 379.9 | 0.24 |
| mp-1077743 | BC ₅ | 382.2 | 377.4 | 0.26 |
| mp-1018649 | BC ₅ | 382.0 | 378.2 | 0.27 |
| oqmd-16166 | BC ₂ N | 409.5 | 377.9 | 0.53 |
| oqmd-16167 | BC ₂ N | 409.9 | 377.6 | 0.53 |

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